

$E$ : PEZZO DIFETOSO  $\rightarrow P(E) = 20\% = 0,20$  costante in ogni prova

$$n = 4$$

$X$ : n° di volte ..  $E$  nelle  $n$  prove INDIPENDENTI  
v.c. Binomiale  $P(E) = 0,20 = p$

$X$  v.c. Bin( $n=4; p=0,2$ )

$$X = 0, 1, 2, 3, 4 \rightarrow f(x) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{4}{x} (0,2)^x (0,8)^{4-x}$$

$$E(X) = np = 4 \cdot (0,2)$$

$$V(X) = np(1-p) = 4 \cdot (0,2) \cdot (0,8)$$

$$a) P(X=1) = f(x=1) = \binom{4}{1} (0,2)^1 (0,8)^3 = 0,4096$$

$$b) P(X=0) = f(x=0) = \binom{4}{0} (0,2)^0 (0,8)^4 = 0,4096$$

$$c) P(X \leq 2) = \sum_{x=0}^2 f(x) = f(x=0) + f(x=1) + f(x=2) = 2 \cdot (0,4096) + \binom{4}{2} (0,2)^2 (0,8)^2 = 0,9728$$

ES. 8

lancio il DADO 5 =  $n$  PROVE INDIPENDENTI

$E$ : [3]  $P(E) = \frac{1}{6} = p$  costante nelle  $n$  PROVE

$X$ : n° di volte ...  $E$  nelle  $n$  prove INDIP.  $X = 0, 1, \dots, 5$   
v.c. Binomiale ( $n=5; p=\frac{1}{6}$ )  
 $f(x) = \binom{5}{x} \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{5-x}$

$$a) P(X=0) = f(x=0) = 0,4019$$

$$b) P(X \geq 1) = 1 - P(X=0) = 0,5981$$

$$c) P(X=4) = f(x=4) = 0,0032$$

ES. 9

$n = 10$  INDIPENDENTI

$$E = \boxed{M} (= F) \quad P(E: M) = P(E: F) = 0,50 = p$$

$X$ : n° di volte che ...  $E$  nelle  $n$  prove INDIP.

v.c. Binomiale ( $n = 10; p = 0,5$ )  $X = 0, 1, 2, \dots, 10$

$$a) P(X=5) = f(x=5) = 0,2461 \quad f(x) = \binom{10}{x} (0,5)^x (0,5)^{10-x}$$

$$b) P(3 \leq X \leq 8) = f(x=3) + f(x=4) + \dots + f(x=8) =$$

$$= 1 - [f(x=0) + f(x=1) + f(x=2) + f(x=9) + f(x=10)] =$$

$$= 0,9346$$

ES. 10

$n = 5$  PROVE INDIP.

$E$ : esecuzione la risposta giusta  $\rightarrow P(E) = \frac{1}{3} = p$  cost.

$X$  v.c. Binomiale: n° di volte ...  $E$  .. nelle  $n$

$$X = 0, 1, \dots, 5 \quad f(x) = \binom{5}{x} \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{5-x}$$

$$E(X) = n \cdot p =$$

$$= 5 \cdot \left(\frac{1}{3}\right) = 1,667$$

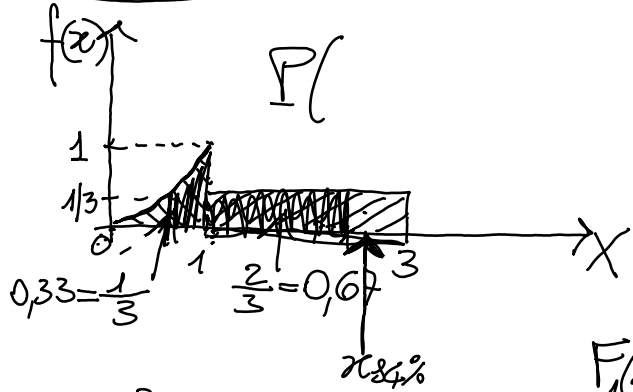
$$P(X \geq 4) = f(x=4) + f(x=5) = 0,04527$$

$$f(x) = \begin{cases} x^2 & (0 < x \leq 1) \\ \frac{k}{3} & (1 < x \leq 3) \\ 0 & \text{ALTRIMENTI} \end{cases}$$

$\Rightarrow k = ? \quad X: \text{U.C. CONTINUA} \quad \int_{\mathbb{R}} f(x) dx = 1$

$$1 = \int_0^1 x^2 dx + \int_1^3 \frac{k}{3} dx = \left[ \frac{x^3}{3} \right]_0^1 + \frac{k}{3} [x]_1^3 = \frac{1}{3} [1^3 - 0] + \frac{k}{3} [3 - 1] = \frac{1}{3} + \frac{2}{3}k = 1 \rightarrow k = 1$$

$$f(x) = \begin{cases} x^2 & ( ) \\ \frac{1}{3} & ( ) \\ 0 & \text{ALTRIMENTI} \end{cases}$$



per  $0 \leq x < 1$

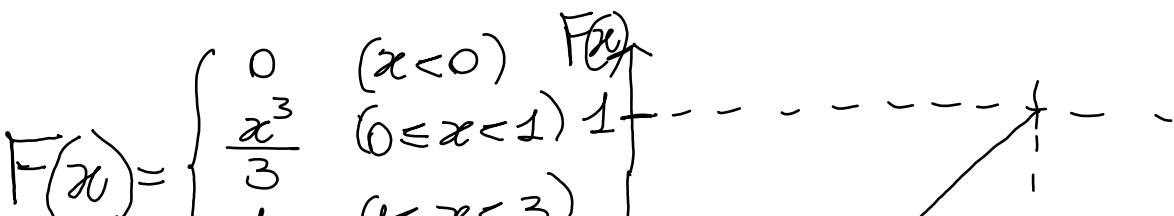
$$F_1(x) = \int_0^x f(t) dt = \int_0^x t^2 dt = \left[ \frac{t^3}{3} \right]_0^x = \frac{1}{3} [x^3 - 0^3] = \frac{1}{3} x^3$$

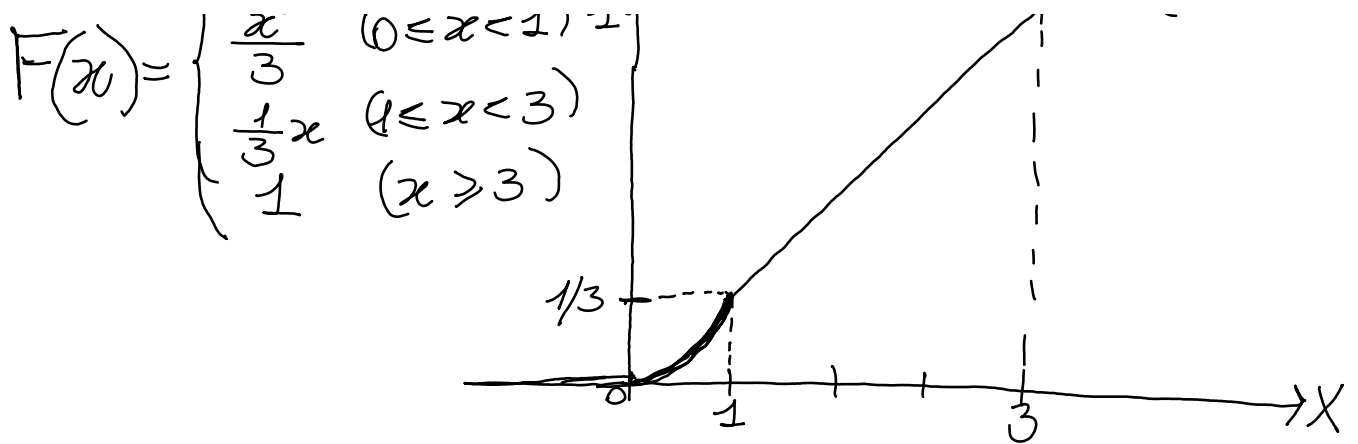
$F_1(x_0) = \int_0^{x_0} f(t) dt = P(X \leq x_0)$   
 $F_1(x_0 = 0.5) = \frac{1}{3} (0.5)^3 = 0.0833 = \frac{1}{12}$

per  $1 \leq x < 3$

$$F_2(x) = \int_0^1 f(t) dt + \int_1^x f(t) dt = \int_0^1 t^2 dt + \int_1^x \frac{1}{3} dt = \frac{1}{3} + \frac{1}{3} [t]_1^x = \frac{1}{3} + \frac{1}{3} [x - 1] = \frac{1}{3} x$$

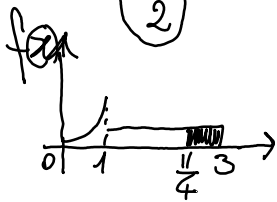
$F(x_0) = P(X \leq x_0) = \int_0^{x_0} f(x) dx$





$$\begin{aligned} P\left(\frac{1}{2} < X < \frac{5}{2}\right) &= P\left(X < \frac{5}{2}\right) - P\left(X < \frac{1}{2}\right) = \\ &= F_2\left(X = \frac{5}{2}\right) - F_1\left(X = \frac{1}{2}\right) = \\ &= \frac{1}{3} \cdot \frac{5}{2} - \frac{1}{3} \left(\frac{1}{2}\right)^3 = \frac{19}{24} = 0,79 \end{aligned}$$

$$\begin{aligned} P\left(X > \frac{11}{4}\right) &= 1 - P\left(X < \frac{11}{4}\right) = 1 - F_2\left(\frac{11}{4}\right) = \\ &= 1 - \frac{1}{3} \cdot \frac{11}{4} = \frac{1}{12} = 0,08 \end{aligned}$$



$$\begin{aligned} E(X) &= \int_{c.e.} x \cdot f(x) dx = \int_0^1 x \cdot (x^2) dx + \int_1^3 x \left(\frac{1}{3}\right) dx = \\ &= \int_0^1 x^3 dx + \int_1^3 \frac{x}{3} dx = \frac{19}{12} \end{aligned}$$

$$\begin{aligned} Y &= 2 + 3X \\ E(Y) &= 2 + 3E(X) = \\ &= 27/4 \end{aligned}$$

~~$P(X \leq x_{0,80}) = 0,80 = F_2(x_{0,80}) = \frac{1}{3} \cdot x_{0,80}$~~

~~$x_{0,80} = (0,80) \cdot 3 = 2,4$~~

$$P(X \leq \underset{\substack{x_{80\%} \\ \cap \\ 1 - .37}}{c}}) = \boxed{0,80} = F_2(x_{80\%}) = \boxed{\frac{1}{3} \cdot x_{80\%}}$$

$$x_{80\%} = 2,4$$

$$x_{80\%} \\ \uparrow \\ [1-3]$$

$$x_{80\%} = 2,4$$

$$P(X \leq x_{50\%}) = 0,50 = F_2(x_{50\%}) = \frac{1}{3} \cdot x_{50\%}$$

$\uparrow$   
 MEDIANA  
 $\uparrow$   
 $[1-3]$

$$x_{50\%} = 1,5$$

$F(x)$

$$a) f(x) = \frac{dF(x)}{dx} <$$

$$f(x) = \begin{cases} \frac{4}{5}x & (0 \leq x < 1) \\ \frac{1}{5} & (1 \leq x < 4) \end{cases}$$

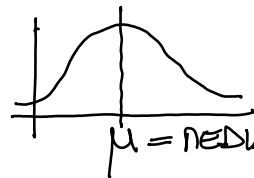
GRAFICO

$$P\left(\frac{1}{2} < X < 2\right) = \frac{1}{2}$$

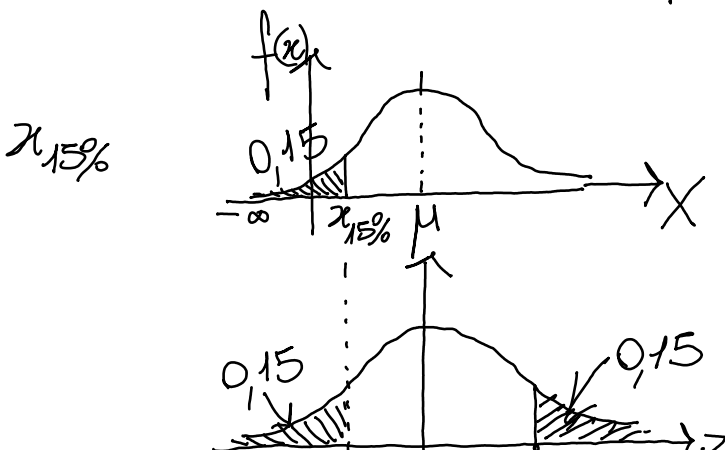
$$E(X) = 1,7667$$

$$x_{84\%} = 3,2$$

v.c.  $N(\mu = 300; \sigma = 15)$



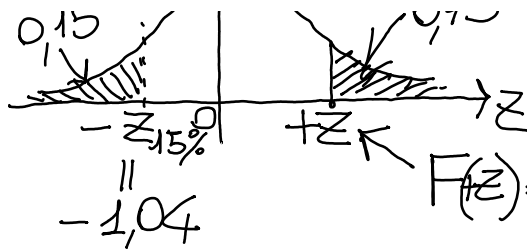
$$\mu = \text{MEDIANA} = \text{MODA} = 300$$



$$Z = \frac{X - \mu_x}{\sigma_x} \sim N(\mu = 0; \sigma^2 = 1)$$

v.c. N STD

TABLE  $Z \rightarrow F(Z_0) = P(Z \leq z_0)$



$$F(z) = P(Z < +z) = 1 - 0,15 = 0,85 \approx 0,8508$$

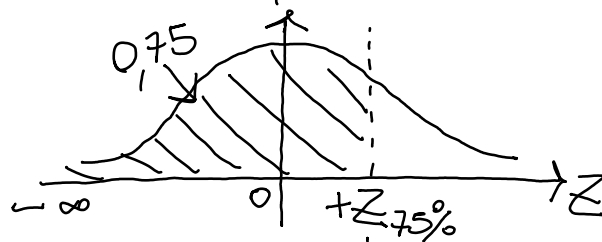
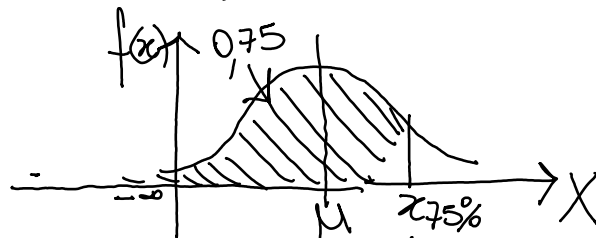
$$z_{15\%} = -1,04 = \frac{x_{15\%} - 300}{15}$$

$$\sigma = 15$$

$$\sigma^2 = (15)^2$$

$$x_{15\%} = 284,4$$

$x_{75\%}$

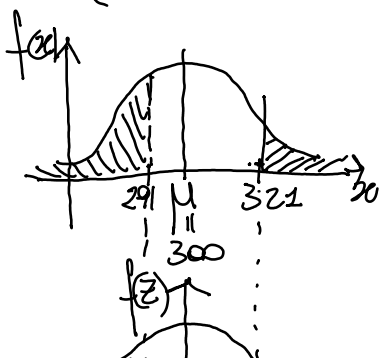


$$F(z_{75\%}) = 0,75 \approx 0,7486$$

$$z_{75\%} = +0,67 = \frac{x_{75\%} - 300}{15}$$

$$q_3 = x_{75\%} = 310,05$$

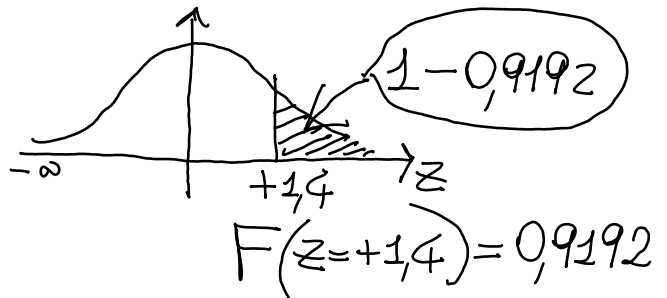
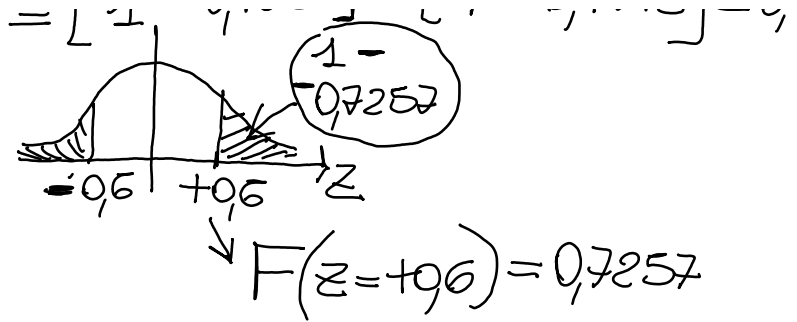
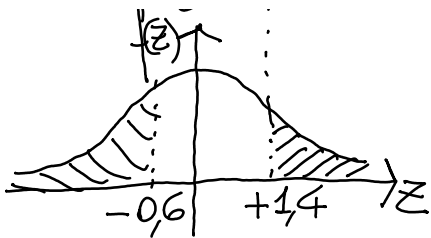
$$P(X < 291 \cup X > 321) = P(X < 291) + P(X > 321) =$$



$$= P\left(z < \frac{291 - 300}{15}\right) + P\left(z > \frac{321 - 300}{15}\right) =$$

$$= P(z < -0,6) + P(z > 1,4) =$$

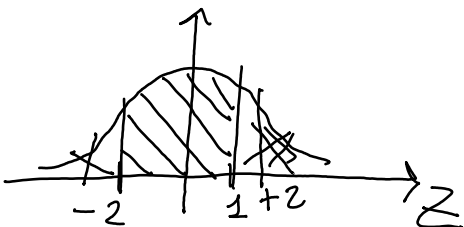
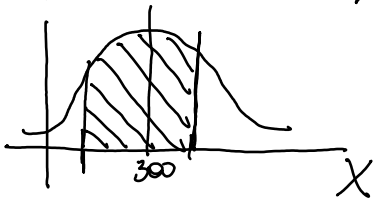
$$= [1 - 0,7257] + [1 - 0,9192] = 0,3551$$



$$P(270 < x < 315) = P\left(\frac{270-300}{15} < z < \frac{315-300}{15}\right) =$$

$$= P(-2 < z < +1) = P(z < +1) - P(z < -2) =$$

$$= 0,8185$$



$$P(X < 400) = P\left(z < \frac{400-300}{15} = +6,67\right) = 1$$

$$P(z < 3,99) = F(3,99) = 1$$

